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FOURIER ANALYSIS BY THE METHOD OF SELECTED ORDINATES

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BY THE
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ABSTRACT

The selected ordinate method for determining the Fourier coefficients of a periodic curve is of great use when an approximate value is desired in a very short time. Tables of the ordinates are given. Evaluation of a coefficient is reduced to a simple summation.

FOURIER ANALYSIS BY THE METHOD OF SELECTED ORDINATES

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SUMMARY

A method is presented for approximating Fourier coefficients to at least two significant figures. By choosing values at selectively weighted intervals of a periodic curve, the sine and cosine harmonics up to $n = 20$ may be approximated in a very short time.

* * * * *

INTRODUCTION

The method of selected ordinates is used to approximate the Fourier coefficients of a periodic curve to at least two significant figures. The selected ordinate method is particularly suited for the determination of sinusoidal errors in such devices as resolvers, gear trains, gyros, and accelerometers; as well as eccentricities of ball bearings; or simply examining the harmonic content of any periodic curve and avoiding a point-by-point numerical integration.

EVALUATION

To evaluate an integral of the form:

$$\int_a^b f(x) g(x) dx$$

the values of $f(x)$ are determined at x -values corresponding to the centers of N successive equal area intervals of a known $g(x)$ and are then summed:

$$\begin{aligned} \int_a^b f(x) g(x) dx &\approx \left\{ \sum_{i=1}^N f(x_i) \right\} \frac{1}{N} \int_a^b |g(x)| dx \\ &= \frac{1}{N} \left\{ \sum_{i=1}^N f(x_i) \right\} \quad (\text{total absolute area}) \end{aligned}$$

which implicitly assumes that the curve has a Fourier expansion. The method is similar to the procedure for evaluating an integral by a sum of trapezoids except that in the selected ordinate case the x-interval varies by a given criterion.

If a function has a Fourier series expansion of period $2L$ and is given in the fundamental interval of 0 to $2L$, it can be represented in the familiar form:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right\}$$

where:

$$a_n = \frac{1}{L} \int_0^{2L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx \quad n = 0, 1, 2, \dots$$

$$b_n = \frac{1}{L} \int_0^{2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx \quad n = 1, 2, 3, \dots$$

(Tables have been prepared for functions of period 2π , henceforth $L=\pi$.) It is apparent from the form of a_n and b_n that the Fourier coefficients can be approximated by the method of selected ordinates:

$$\frac{1}{\pi} \int_0^{2\pi} f(x) g(x) dx \approx \frac{1}{\pi} \left\{ \sum_{i=1}^N f(x_i) \right\} \frac{\text{total absolute area}}{N}$$

In the case of Fourier analysis, the total absolute area is the absolute area under the sine or cosine harmonic curve. The absolute area is equal to 4 for all positive $n>0$ and is equal to 2π for $n=0$. Thus the Fourier coefficient is evaluated by the expression:

$$\frac{4}{\pi} \sum_{i=1}^N f(x_i) \frac{1}{N} = \frac{4}{\pi} \frac{\text{sum}}{\text{division factor}} \quad (n = 1, 2, 3, \dots)$$

$$2 \sum_{i=1}^N f(x_i) \frac{1}{N} = 2 \frac{\text{sum}}{\text{division factor}} \quad (n = 0)$$

To determine the sum, add the values of $f(x_i)$ for each selected ordinate indicated in the appropriate table; the sign

of each term is given by the product of the sign of $f(x_i)$ and the sign associated with the selected ordinate. If in determining the sum, the value of $f(x)$ at a selected ordinate is a point of discontinuity, the Dirichlet condition is used:

$$f(x_i) = \lim_{\epsilon \rightarrow 0} \frac{1}{2} \{ f(x_i + \epsilon) + f(x_i - \epsilon) \}$$

The division factor is sometimes not equal to N , the number of selected ordinates, if the calculated selected ordinate occurs where the sine or cosine harmonic curve crosses the axis. This is indicated in the accompanying tables by not associating a sign with that ordinate. Since the calculated ordinate will have equal positive and negative area contributions, the value of the division factor is given at the bottom of each table for convenience.

The accuracy of the selected ordinate method is proportional to the number of selected ordinates used. Some applications may warrant the use of less than 90 ordinates; the tables can be subdivided into 30, 18, 10, 6, or 2 ordinates where the first ordinate will be the second, third, fifth, eighth, or twenty-third ordinate, respectively, of the tabulated 90.

The selected ordinate method for determining the Fourier coefficients of a periodic curve has proved to be most valuable when an approximate value is required in a short time.

EXAMPLE

As an example of the use and accuracy of the selected ordinate method as applied to Fourier analysis, the expansion for:

$$f(x) = \begin{cases} 1, & \frac{2\pi}{3} \leq x \leq \pi \\ -1, & \pi \leq x \leq \frac{4\pi}{3} \\ 0, & \text{otherwise} \end{cases}$$

will be determined. Using the tables in the appendix:

$$\begin{aligned} a_0: \quad \text{sum} &= 30(0) + 15(+1) - 15(+1) + 30(0) = 0 \\ \therefore a_0 &= 0 \end{aligned}$$

$$a_1: \text{ sum} = 25(0) - 19(+1) - 19(-1) + 25(0) = 0$$

$$\therefore a_1 = 0$$

and so on for the other a_n as $f(x)$ is an odd function:

$$b_1: \text{ sum} = 33(0) + 12(+1) - 11(-1) + 34(0) = 23$$

$$\therefore b_1 = \frac{4}{\pi} \frac{(23)}{90} = \frac{1}{\pi} (1.02)$$

$$b_2: \text{ sum} = 27(0) + 17(+1) - 17(-1) + 27(0) = -34$$

$$\therefore b_2 = \frac{4}{\pi} \frac{(-34)}{88} = -\frac{1}{\pi} (1.545)$$

$$b_{15}: \text{ sum} = 30(0) + 3(1) - 3(1) - 3(1) + 3(1) - 3(-1) + 3(-1) \\ - 3(-1) + 3(-1) - 3(-1) = 6$$

$$\therefore b_{15} = \frac{4}{\pi} \frac{(6)}{90} = \frac{1}{\pi} (0.267)$$

By a direct integration:

$$f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\cos\left(\frac{n\pi}{3}\right) - 1 \right) \sin(nx)$$

$$= \frac{1}{\pi} (1.000) \sin(x) - \frac{1}{\pi} (1.500) \sin(2x)$$

$$+ \dots + \frac{1}{\pi} (0.267) \sin(15x) + \dots$$

whereas the selected ordinate method gives:

$$f(x) = \frac{1}{\pi} (1.022) \sin (x) - \frac{1}{\pi} (1.545) \sin (2x) \\ + \dots + \frac{1}{\pi} (0.267) \sin (15x) + \dots$$

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National Aeronautics and Space Administration
Electronics Research Center
Cambridge, Massachusetts, October 1968
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90 SELECTED ORDINATES FOR
SIN(0)

12.11 +	92.55 +	192.11 -	272.55 -
21.04 +	95.10 +	201.04 -	275.10 -
27.27 +	97.67 +	207.27 -	277.67 -
32.39 +	100.25 +	212.39 -	280.25 -
36.87 +	102.84 +	216.87 -	282.84 -
40.93 +	105.47 +	220.93 -	285.47 -
44.68 +	108.13 +	224.68 -	288.13 -
48.19 +	110.83 +	228.19 -	290.83 -
51.53 +	113.58 +	231.53 -	293.58 -
54.71 +	116.39 +	234.71 -	296.39 -
57.77 +	119.27 +	237.77 -	299.27 -
60.74 +	122.24 +	240.74 -	302.24 -
63.62 +	125.30 +	243.62 -	305.30 -
66.43 +	128.48 +	246.43 -	308.48 -
69.18 +	131.82 +	249.18 -	311.82 -
71.88 +	135.33 +	251.88 -	315.33 -
74.54 +	139.08 +	254.54 -	319.08 -
77.17 +	143.14 +	257.17 -	323.14 -
79.76 +	147.62 +	259.76 -	327.62 -
82.34 +	152.74 +	262.34 -	332.74 -
84.91 +	158.97 +	264.91 -	338.97 -
87.46 +	167.90 +	267.46 -	347.90 -
90.00 +		270.00 -	

DIVISION FACTOR = 90

90 SELECTED ORDINATES FOR
SIN(20)

8.58 +	102.18 -	188.58 +	282.18 -
14.97 +	107.35 -	194.97 +	287.35 -
19.48 +	111.42 -	199.48 +	291.42 -
23.23 +	114.94 -	203.23 +	294.94 -
26.57 +	118.13 -	206.57 +	298.13 -
29.64 +	121.10 -	209.64 +	301.10 -
32.52 +	123.91 -	212.52 +	303.91 -
35.27 +	126.61 -	215.27 +	306.61 -
37.93 +	129.24 -	217.93 +	309.24 -
40.53 +	131.82 -	220.53 +	311.82 -
43.09 +	134.37 -	223.09 +	314.37 -
45.64 +	136.92 -	225.64 +	316.92 -
48.19 +	139.48 -	228.19 +	319.48 -
50.77 +	142.08 -	230.77 +	322.08 -
53.40 +	144.74 -	233.40 +	324.74 -
56.10 +	147.49 -	236.10 +	327.49 -
58.91 +	150.37 -	238.91 +	330.37 -
61.88 +	153.44 -	241.88 +	333.44 -
65.07 +	156.78 -	245.07 +	336.78 -
68.59 +	160.53 -	248.59 +	340.53 -
72.66 +	165.04 -	252.66 +	345.04 -
77.83 +	171.43 -	257.83 +	351.43 -
90.00		270.00	

DIVISION FACTOR = 88

APPENDIX

90 SELECTED ORDINATES FOR
SIN(30)

7.02 +	92.56 -	187.02 -	272.56 +
12.29 +	95.16 -	192.29 -	275.16 +
16.07 +	97.86 -	196.07 -	277.86 +
19.26 +	100.75 -	199.26 -	280.75 +
22.15 +	103.94 -	202.15 -	283.94 +
24.85 +	107.72 -	204.85 -	287.72 +
27.45 +	112.99 -	207.45 -	292.99 +
30.00 +	127.02 +	210.00 -	307.02 -
32.56 +	132.29 +	212.56 -	312.29 -
35.16 +	136.07 +	215.16 -	316.07 -
37.86 +	139.26 +	217.86 -	319.26 -
40.75 +	142.15 +	220.75 -	322.15 -
43.94 +	144.85 +	223.94 -	324.85 -
47.72 +	147.45 +	227.72 -	327.45 -
52.99 +	150.00 +	232.99 -	330.00 -
67.02 -	152.56 +	247.02 +	332.56 -
72.29 -	155.16 +	252.29 +	335.16 -
76.07 -	157.86 +	256.07 +	337.86 -
79.26 -	160.75 +	259.26 +	340.75 -
82.15 -	163.94 +	262.15 +	343.94 -
84.85 -	167.72 +	264.85 +	347.72 -
87.45 -	172.99 +	267.45 +	352.99 -
90.00 -		270.00 +	

DIVISION FACTOR = 90

90 SELECTED ORDINATES FOR
SIN(40)

6.09 +	98.68 +	186.09 +	278.68 +
10.71 +	102.47 +	190.71 +	282.47 +
14.07 +	105.55 +	194.07 +	285.55 +
16.96 +	108.31 +	196.96 +	288.31 +
19.62 +	110.91 +	199.62 +	290.91 +
22.19 +	113.46 +	202.19 +	293.46 +
24.74 +	116.04 +	204.74 +	296.04 +
27.37 +	118.75 +	207.37 +	298.75 +
30.19 +	121.72 +	210.19 +	301.72 +
33.39 +	125.27 +	213.39 +	305.27 +
37.52 +	130.72 +	217.52 +	310.72 +
49.29 -	142.49 -	229.29 -	322.49 -
54.74 -	146.62 -	234.74 -	326.62 -
58.29 -	149.82 -	238.29 -	329.82 -
61.26 -	152.64 -	241.26 -	332.64 -
63.97 -	155.27 -	243.97 -	335.27 -
66.55 -	157.82 -	246.55 -	337.82 -
69.10 -	160.39 -	249.10 -	340.39 -
71.70 -	163.05 -	251.70 -	343.05 -
74.46 -	165.94 -	254.46 -	345.94 -
77.54 -	169.30 -	257.54 -	349.30 -
81.33 -	173.92 -	261.33 -	353.92 -
90.00		270.00	

DIVISION FACTOR = 88

90 SELECTED ORDINATES FOR
SIN(50)

5.46 +	92.57 +	185.46 -	272.57 -
9.64 +	95.28 +	189.64 -	275.28 -
12.73 +	98.37 +	192.73 -	278.37 -
15.44 +	102.55 +	195.44 -	282.55 -
18.00 +	113.46 -	198.00 -	293.46 +
20.57 +	117.64 -	200.57 -	297.64 +
23.28 +	120.73 -	203.28 -	300.73 +
26.37 +	123.44 -	206.37 -	303.44 +
30.55 +	126.00 -	210.55 -	306.00 +
41.46 -	128.57 -	221.46 +	308.57 +
45.64 -	131.28 -	225.64 +	311.28 +
48.73 -	134.37 -	228.73 +	314.37 +
51.44 -	138.55 -	231.44 +	318.55 +
54.00 -	149.46 +	234.00 +	329.46 -
56.57 -	153.64 +	236.57 +	333.64 -
59.28 -	156.73 +	239.28 +	336.73 -
62.37 -	159.44 +	242.37 +	339.44 -
66.55 -	162.00 +	246.55 +	342.00 -
77.46 +	164.57 +	257.46 -	344.57 -
81.64 +	167.28 +	261.64 -	347.28 -
84.73 +	170.37 +	264.73 -	350.37 -
87.44 +	174.55 +	267.44 -	354.55 -
90.00 +		270.00 -	

DIVISION FACTOR = 90

90 SELECTED ORDINATES FOR
SIN(60)

4.99 +	97.14 -	184.99 +	277.14 -
8.86 +	100.37 -	188.86 +	280.37 -
11.76 +	103.08 -	191.76 +	283.08 -
14.37 +	105.64 -	194.37 +	285.64 -
16.93 +	108.25 -	196.93 +	288.25 -
19.64 +	111.15 -	199.64 +	291.15 -
22.87 +	115.02 -	202.87 +	295.02 -
30.00	124.99 +	210.00	304.99 +
37.14 -	128.86 +	217.14 -	308.86 +
40.37 -	131.76 +	220.37 -	311.76 +
43.08 -	134.37 +	223.08 -	314.37 +
45.64 -	136.93 +	225.64 -	316.93 +
48.25 -	139.64 +	228.25 -	319.64 +
51.15 -	142.87 +	231.15 -	322.87 +
55.02 -	150.00	235.02 -	330.00
64.99 +	157.14 -	244.99 +	337.14 -
68.86 +	160.37 -	248.86 +	340.37 -
71.76 +	163.08 -	251.76 +	343.08 -
74.37 +	165.64 -	254.37 +	345.64 -
76.93 +	168.25 -	256.93 +	348.25 -
79.64 +	171.15 -	259.64 +	351.15 -
82.87 +	175.02 -	262.87 +	355.02 -
90.00		270.00	

DIVISION FACTOR = 84

90 SELECTED ORDINATES FOR

SIN(70)

4.63 +	92.59 -	184.63 -	272.59 +
8.26 +	95.50 -	188.26 -	275.50 +
11.03 +	99.86 -	191.03 -	279.86 +
13.59 +	108.71 +	193.59 -	288.71 -
16.23 +	111.95 +	196.23 -	291.95 -
19.34 +	114.62 +	199.34 -	294.62 -
27.45 -	117.18 +	207.45 +	297.18 -
32.60 -	119.90 +	212.60 +	299.90 -
35.60 -	123.31 +	215.60 +	303.31 -
38.21 -	132.47 -	218.21 +	312.47 +
40.79 -	136.39 -	220.79 +	316.39 +
43.62 -	139.22 -	223.62 +	319.22 +
47.54 -	141.80 -	227.54 +	321.80 +
56.70 +	144.41 -	236.70 -	324.41 +
60.11 +	147.41 -	240.11 -	327.41 +
62.83 +	152.56 -	242.83 -	332.56 +
65.39 +	160.67 +	245.39 -	340.67 -
68.06 +	163.78 +	248.06 -	343.78 -
71.30 +	166.42 +	251.30 -	346.42 -
80.15 -	168.98 +	260.15 +	348.98 -
84.51 -	171.75 +	264.51 +	351.75 -
87.42 -	175.38 +	267.42 +	355.38 -
90.00 -		270.00 +	

DIVISION FACTOR = 90

90 SELECTED ORDINATES FOR

SIN(86)

4.34 +	96.24 +	184.34 +	276.24 +
7.78 +	99.16 +	187.78 +	279.16 +
10.46 +	101.73 +	190.46 +	281.73 +
13.02 +	104.38 +	193.02 +	284.38 +
15.86 +	107.64 +	195.86 +	287.64 +
20.36 +	116.25 -	200.36 +	296.25 -
28.31 -	119.91 -	208.31 -	299.91 -
31.32 -	122.64 -	211.32 -	302.64 -
33.91 -	125.20 -	213.91 -	305.20 -
36.53 -	127.97 -	216.53 -	307.97 -
39.65 -	131.96 -	219.65 -	311.96 -
48.05 +	140.36 +	228.05 +	320.36 +
52.04 +	143.48 +	232.04 +	323.48 +
54.81 +	146.10 +	234.81 +	326.10 +
57.37 +	148.69 +	237.37 +	328.69 +
60.10 +	151.70 +	240.10 +	331.70 +
63.76 +	159.65 -	243.76 +	339.65 -
72.37 -	164.15 -	252.37 -	344.15 -
75.63 -	166.99 -	255.63 -	346.99 -
78.28 -	169.55 -	258.28 -	349.55 -
80.85 -	172.23 -	260.85 -	352.23 -
83.77 -	175.67 -	263.77 -	355.67 -
90.00		270.00	

DIVISION FACTOR = 86

90 SELECTED ORDINATES FOR
SIN(90)

4.10 +	92.62 +	184.10 -	272.62 -
7.39 +	95.91 +	187.39 -	275.91 -
10.00 +	104.10 -	190.00 -	284.10 +
12.62 +	107.39 -	192.62 -	287.39 +
15.91 +	110.00 -	195.91 -	290.00 +
24.10 -	112.62 -	204.10 +	292.62 +
27.39 -	115.91 -	207.39 +	295.91 +
30.00 -	124.10 +	210.00 +	304.10 -
32.62 -	127.39 +	212.62 +	307.39 -
35.91 -	130.00 +	215.91 +	310.00 -
44.10 +	132.62 +	224.10 -	312.62 -
47.39 +	135.91 +	227.39 -	315.91 -
50.00 +	144.10 -	230.00 -	324.10 +
52.62 +	147.89 -	232.62 -	327.39 +
55.91 +	150.00 -	235.91 -	330.00 +
64.10 -	152.62 -	244.10 +	332.62 +
67.39 -	155.91 -	247.39 +	335.91 +
70.00 -	164.10 +	250.00 +	344.10 -
72.62 -	167.39 +	252.62 +	347.39 -
75.91 -	170.00 +	255.91 +	350.00 -
84.10 +	172.62 +	264.10 -	352.62 -
87.39 +	175.91 +	267.39 -	355.91 -
90.00 +		270.00 -	

DIVISION FACTOR = 90

90 SELECTED ORDINATES FOR
SIN(100)

3.90 +	95.63 -	183.90 +	275.63 -
7.06 +	98.37 -	187.06 +	278.37 -
9.64 +	100.95 -	189.64 +	280.95 -
12.38 +	104.11 -	192.38 +	284.11 -
18.00	111.90 +	198.00	291.90 +
23.63 -	115.06 +	203.63 -	295.06 +
26.37 -	117.64 +	206.37 -	297.64 +
28.95 -	120.38 +	208.95 -	300.38 +
32.11 -	126.00	212.11 -	306.00
39.90 +	131.63 -	219.90 +	311.63 -
43.06 +	134.37 -	223.06 +	314.37 -
45.64 +	136.95 -	225.64 +	316.95 -
48.38 +	140.11 -	228.38 +	320.11 -
54.00	147.90 +	234.00	327.90 +
59.63 -	151.06 +	239.63 -	331.06 +
62.37 -	153.64 +	242.37 -	333.64 +
64.95 -	156.38 +	244.95 -	336.38 +
68.11 -	162.00	248.11 -	342.00
75.90 +	167.63 -	255.90 +	347.63 -
79.06 +	170.37 -	259.06 +	350.37 -
81.64 +	172.95 -	261.64 +	352.95 -
84.38 +	176.11 -	264.38 +	356.11 -
90.00		270.00	

DIVISION FACTOR = 80

90 SELECTED ORDINATES FOR
SIN(110)

3.73 +	92.67 -	183.73 -	272.67 +
6.78 +	97.09 -	186.78 -	277.09 +
9.35 +	103.44 +	189.35 -	283.44 -
12.31 +	106.14 +	192.31 -	286.14 -
19.72 -	108.77 +	199.72 +	288.77 -
22.90 -	112.64 +	202.90 +	292.64 -
25.48 -	119.52 -	205.48 +	299.52 +
28.35 -	122.27 -	208.35 +	302.27 +
35.68 +	124.88 -	215.68 -	304.88 +
39.02 +	128.44 -	219.02 -	308.44 +
41.61 +	135.60 +	221.61 -	315.60 -
44.41 +	138.40 +	224.41 -	318.40 -
51.57 -	140.99 +	231.57 +	320.99 -
55.13 -	144.33 +	235.13 +	324.33 -
57.74 -	151.66 -	237.74 +	331.66 +
60.49 -	154.53 -	240.49 +	334.53 +
67.37 +	157.11 -	247.37 -	337.11 +
71.24 +	160.29 -	251.24 -	340.29 +
73.87 +	167.70 +	253.87 -	347.70 -
76.57 +	170.66 +	256.57 -	350.66 -
82.92 -	173.23 +	262.92 +	353.23 -
87.34 -	176.28 +	267.34 +	356.28 -
90.00 -		270.00 +	

DIVISION FACTOR = 90

90 SELECTED ORDINATES FOR
SIN(120)

3.57 +	95.19 +	183.57 +	275.19 +
6.54 +	97.82 +	186.54 +	277.82 +
9.13 +	100.58 +	189.13 +	280.58 +
12.51 +	107.50 -	192.51 +	287.50 -
19.43 -	110.88 -	199.43 -	290.88 -
22.19 -	113.47 -	202.19 -	293.47 -
24.82 -	116.44 -	204.82 -	296.44 -
30.00	123.57 +	210.00	303.57 +
35.19 +	126.54 +	215.19 +	306.54 +
37.82 +	129.13 +	217.82 +	309.13 +
40.58 +	132.51 +	220.58 +	312.51 +
47.50 -	139.43 -	227.50 -	319.43 -
50.88 -	142.19 -	230.88 -	322.19 -
53.47 -	144.82 -	233.47 -	324.82 -
56.44 -	150.00	236.44 -	330.00
63.57 +	155.19 +	243.57 +	335.19 +
66.54 +	157.82 +	246.54 +	337.82 +
69.13 +	160.58 +	249.13 +	340.58 +
72.51 +	167.50 -	252.51 +	347.50 -
79.43 -	170.88 -	259.43 -	350.88 -
82.19 -	173.47 -	262.19 -	353.47 -
84.82 -	176.44 -	264.82 -	356.44 -
90.00		270.00	

DIVISION FACTOR = 84

90 SELECTED ORDINATES FOR

SIN(130)

3.44 +	92.72 +	183.44 -	272.72 -
6.34 +	99.42 -	186.34 -	279.42 +
8.96 +	102.66 -	188.96 -	282.66 +
14.78 -	105.25 -	194.78 +	285.25 +
18.96 -	108.68 -	198.96 +	288.68 +
21.56 -	115.22 +	201.56 +	295.22 -
24.55 -	117.89 +	204.55 +	297.89 -
31.40 +	120.66 +	211.40 -	300.66 -
34.23 +	127.46 -	214.23 -	307.46 +
36.87 +	130.56 -	216.87 -	310.56 +
43.16 -	133.15 -	223.16 +	313.15 +
46.86 -	136.85 -	226.86 +	316.85 +
49.45 -	143.14 +	229.45 +	323.14 -
52.55 -	145.78 +	232.55 +	325.78 -
59.35 +	148.61 +	239.35 -	328.61 -
62.12 +	155.46 -	242.12 -	335.46 +
64.79 +	158.45 -	244.79 -	338.45 +
71.33 -	161.05 -	251.33 +	341.05 +
74.76 -	165.23 -	254.76 +	345.23 +
77.35 -	171.05 +	257.35 +	351.05 -
80.59 -	173.67 +	260.59 +	353.67 -
87.29 +	176.57 +	267.29 -	356.57 -
90.00 +		270.00 -	

DIVISION FACTOR = 90

90 SELECTED ORDINATES FOR

SIN(140)

3.32 +	94.85 -	183.32 +	274.85 -
6.16 +	97.44 -	186.16 +	277.44 -
8.84 +	100.72 -	188.84 +	280.72 -
15.34 -	107.10 +	195.34 -	287.10 +
18.47 -	109.75 +	198.47 -	289.75 +
21.07 -	112.66 +	201.07 -	292.66 +
26.94 +	119.28 -	206.94 +	299.28 -
30.76 +	122.06 -	210.76 +	302.06 -
33.35 +	124.78 -	213.35 +	304.78 -
36.84 +	131.36 +	216.84 +	311.36 +
43.02 -	134.37 +	223.02 -	314.37 +
45.64 -	136.99 +	225.64 -	316.99 +
48.65 -	143.17 -	228.65 -	323.17 -
55.23 +	146.66 -	235.23 +	326.66 -
57.95 +	149.25 -	237.95 +	329.25 -
60.73 +	153.07 -	240.73 +	333.07 -
67.35 -	158.94 +	247.35 -	338.94 +
70.26 -	161.54 +	250.26 -	341.54 +
72.91 -	164.67 +	252.91 -	344.67 +
79.29 +	171.17 -	259.29 +	351.17 -
82.57 +	173.85 -	262.57 +	353.85 -
85.16 +	176.69 -	265.16 +	356.69 -
90.00		270.00	

DIVISION FACTOR = 88

90 SELECTED ORDINATES FOR
SIN(150)

3.22 +	92.79 -	183.22 -	272.79 +
6.00 +	99.22 +	186.00 -	279.22 -
8.79 +	102.00 +	188.79 -	282.00 -
15.22 -	104.79 +	195.22 +	284.79 -
18.00 -	111.22 -	198.00 +	291.22 +
20.79 -	114.00 -	200.79 +	294.00 +
27.22 +	116.79 -	207.22 -	296.79 +
30.00 +	123.22 +	210.00 -	303.22 -
32.79 +	126.00 +	212.79 -	306.00 -
39.22 -	128.79 +	219.22 +	308.79 -
42.00 -	135.22 -	222.00 +	315.22 +
44.79 -	138.00 -	224.79 +	318.00 +
51.22 +	140.79 -	231.22 -	320.79 +
54.00 +	147.22 +	234.00 -	327.22 -
56.79 +	150.00 +	236.79 -	330.00 -
63.22 -	152.79 +	243.22 +	332.79 -
66.00 -	159.22 -	246.00 +	339.22 +
68.79 -	162.00 -	248.79 +	342.00 +
75.22 +	164.79 -	255.22 -	344.79 +
78.00 +	171.22 +	258.00 -	351.22 -
80.79 +	174.00 +	260.79 -	354.00 -
87.22 -	176.79 +	267.22 +	356.79 -
90.00 -		270.00 +	

DIVISION FACTOR = 90

90 SELECTED ORDINATES FOR
SIN(160)

3.12 +	94.58 +	183.12 +	274.58 +
5.87 +	97.19 +	185.87 +	277.19 +
8.82 +	103.13 -	188.82 +	283.13 -
14.96 -	106.32 -	194.96 -	286.32 -
17.60 -	108.99 -	197.60 -	288.99 -
20.98 -	115.18 +	200.98 -	295.18 +
26.74 +	118.05 +	206.74 +	298.05 +
29.35 +	120.85 +	209.35 +	300.85 +
34.83 -	127.08 -	214.83 -	307.08 -
38.50 -	129.78 -	218.50 -	309.78 -
41.12 -	132.84 -	221.12 -	312.84 -
47.17 +	138.89 +	227.17 +	318.89 +
50.23 +	141.51 +	230.23 +	321.51 +
52.93 +	145.18 +	232.93 +	325.18 +
59.16 -	150.66 -	239.16 -	330.66 -
61.96 -	153.27 -	241.96 -	333.27 -
64.83 -	159.03 +	244.83 -	339.03 +
71.02 +	162.41 +	251.02 +	342.41 +
73.69 +	165.05 +	253.69 +	345.05 +
76.88 +	171.19 -	256.88 +	351.19 -
82.82 -	174.14 -	262.82 -	354.14 -
85.43 -	176.89 -	265.43 -	356.89 -
90.00		270.00	

DIVISION FACTOR = 88

90 SELECTED ORDINATES FOR
SIN(170)

3.04 +	92.89 +	183.04 -	272.89 -
5.75 +	98.87 -	185.75 -	278.87 +
8.99 +	101.50 -	188.99 -	281.50 +
14.66 -	106.60 +	194.66 +	286.60 -
17.27 -	110.43 +	197.27 +	290.43 -
23.09 +	113.08 +	203.09 -	293.08 -
26.18 +	119.10 -	206.18 -	299.10 +
28.94 +	121.92 -	208.94 -	301.92 +
34.99 -	124.90 -	214.99 +	304.90 +
37.67 -	130.81 +	217.67 +	310.81 -
41.12 -	133.42 +	221.12 +	313.42 -
46.59 +	138.89 -	226.59 -	318.89 +
49.20 +	142.34 -	229.20 -	322.34 +
55.11 -	145.02 -	235.11 +	325.02 +
58.09 -	151.07 +	238.09 +	331.07 -
60.91 -	153.83 +	240.91 +	333.83 -
66.93 +	156.92 +	246.93 -	336.92 -
69.58 +	162.74 -	249.58 -	342.74 +
73.41 +	165.35 -	253.41 -	345.35 +
78.51 -	171.02 +	258.51 +	351.02 -
81.14 -	174.26 +	261.14 +	354.26 -
87.12 +	176.97 +	267.12 -	356.97 -
90.00 +		270.00 -	

DIVISION FACTOR = 90

90 SELECTED ORDINATES FOR
SIN(180)

2.96 +	94.36 -	182.96 +	274.36 -
5.65 +	97.05 -	185.65 +	277.05 -
10.00	102.96 +	190.00	282.96 +
14.36 -	105.65 +	194.36 -	285.65 +
17.05 -	110.00	197.05 -	290.00
22.96 +	114.36 -	202.96 +	294.36 -
25.65 +	117.05 -	205.65 +	297.05 -
30.00	122.96 +	210.00	302.96 +
34.36 -	125.65 +	214.36 -	305.65 +
37.05 -	130.00	217.05 -	310.00
42.96 +	134.36 -	222.96 +	314.36 -
45.65 +	137.05 -	225.65 +	317.05 -
50.00	142.96 +	230.00	322.96 +
54.36 -	145.65 +	234.36 -	325.65 +
57.05 -	150.00	237.05 -	330.00
62.96 +	154.36 -	242.96 +	334.36 -
65.65 +	157.05 -	245.65 +	337.05 -
70.00	162.96 +	250.00	342.96 +
74.36 -	165.65 +	254.36 -	345.65 +
77.05 -	170.00	257.05 -	350.00
82.96 +	174.36 -	262.96 +	354.36 -
85.65 +	177.05 -	265.65 +	357.05 -
90.00		270.00	

DIVISION FACTOR = 72

90 SELECTED ORDINATES FOR SIN(190)				90 SELECTED ORDINATES FOR SIN(200)			
2.88 +	93.04 -	182.88 -	273.04 +	2.82 +	94.19 +	182.82 +	274.19 +
5.56 +	98.52 +	185.56 -	278.52 -	5.48 +	97.06 +	185.48 +	277.06 +
10.91 -	101.18 +	190.91 +	281.18 -	10.95 -	102.53 -	190.95 -	282.53 -
14.08 -	106.93 -	194.08 +	286.93 +	13.82 -	105.19 -	193.82 -	285.19 -
17.01 -	109.63 -	197.01 +	289.63 +	18.00	110.82 +	198.00	290.82 +
22.59 +	114.80 +	202.59 -	294.80 -	22.19 +	113.48 +	202.19 +	293.48 +
25.23 +	118.16 +	205.23 -	298.16 -	25.06 +	118.95 -	205.06 +	298.95 -
30.96 -	121.01 +	210.96 +	301.01 -	30.53 -	121.82 -	210.53 -	301.82 -
33.70 -	126.66 -	213.70 +	306.66 +	33.19 -	126.00	213.19 -	306.00
38.54 +	129.29 -	218.54 -	309.29 +	38.82 +	130.19 +	218.82 +	310.19 +
42.23 +	134.99 +	222.23 -	314.99 -	41.48 +	133.06 +	221.48 +	313.06 +
45.02 +	137.78 +	225.02 -	317.78 -	46.95 -	138.53 -	226.95 -	318.53 -
50.72 -	141.47 +	230.72 +	321.47 -	49.82 -	141.19 -	229.82 -	321.19 -
53.35 -	146.31 -	233.35 +	326.31 +	54.00	146.82 +	234.00	326.82 +
59.00 +	149.05 -	239.00 -	329.05 +	58.11 +	149.48 +	238.19 +	329.48 +
61.85 +	154.78 +	241.85 -	334.78 -	61.06 +	154.95 -	241.06 +	334.95 -
65.21 +	157.42 +	245.21 -	337.42 -	66.53 -	157.82 -	246.53 -	337.82 -
70.38 -	163.00 -	250.38 +	343.00 +	69.19 -	162.00	249.19 -	342.00
73.08 -	165.93 -	253.08 +	345.93 +	74.82 +	166.19 +	254.82 +	346.19 +
78.83 +	169.10 -	258.83 -	349.10 +	77.48 +	169.06 +	257.48 +	349.06 +
81.49 +	174.45 +	261.49 -	354.45 -	82.95 -	174.53 -	262.95 -	354.53 -
86.97 -	177.13 +	266.97 +	357.13 -	85.82 -	177.19 -	265.82 -	357.19 -
90.00 -		270.00 +		90.00		270.00	

DIVISION FACTOR = 90

DIVISION FACTOR = 80

90 SELECTED ORDINATES FOR
COS(θ)

1.28 +	107.15 -	181.28 -	287.15 +
3.83 +	114.35 -	183.83 -	294.35 +
6.38 +	119.93 -	186.38 -	299.93 +
8.95 +	124.70 -	188.95 -	304.70 +
11.54 +	128.95 -	191.54 -	308.95 +
14.15 +	132.84 -	194.15 -	312.84 +
16.80 +	136.46 -	196.80 -	316.46 +
19.48 +	139.88 -	199.48 -	319.88 +
22.20 +	143.14 -	202.20 -	323.14 +
24.98 +	146.26 -	204.98 -	326.26 +
27.82 +	149.27 -	207.82 -	329.27 +
30.74 +	152.19 -	210.74 -	332.19 +
33.75 +	155.03 -	213.75 -	335.03 +
36.87 +	157.81 -	216.87 -	337.81 +
40.13 +	160.53 -	220.13 -	340.53 +
43.55 +	163.21 -	223.55 -	343.21 +
47.17 +	165.86 -	227.17 -	345.86 +
51.06 +	168.47 -	231.06 -	348.47 +
55.31 +	171.06 -	235.31 -	351.06 +
60.08 +	173.63 -	240.08 -	353.63 +
65.66 +	176.18 -	245.66 -	356.18 +
72.86 +	178.73 -	252.86 -	358.73 +
90.00		270.00	

DIVISION FACTOR = 88

90 SELECTED ORDINATES FOR
COS(2θ)

1.92 +	93.19 -	181.92 +	273.19 -
4.48 +	95.77 -	184.48 +	275.77 -
7.08 +	98.40 -	187.08 +	278.40 -
9.74 +	101.10 -	189.74 +	281.10 -
12.49 +	103.91 -	192.49 +	283.91 -
15.37 +	106.88 -	195.37 +	286.88 -
18.44 +	110.07 -	198.44 +	290.07 -
21.78 +	113.59 -	201.78 +	293.59 -
25.53 +	117.66 -	205.53 +	297.66 -
30.04 +	122.83 -	210.04 +	302.83 -
36.43 +	135.00	216.43 +	315.00
53.58 -	147.18 +	233.58 -	327.18 +
59.97 -	152.35 +	239.97 -	332.35 +
64.48 -	156.42 +	244.48 -	336.42 +
68.23 -	159.94 +	248.23 -	339.94 +
71.57 -	163.13 +	251.57 -	343.13 +
74.64 -	166.10 +	254.64 -	346.10 +
77.52 -	168.91 +	257.52 -	348.91 +
80.27 -	171.61 +	260.27 -	351.61 +
82.93 -	174.24 +	262.93 -	354.24 +
85.53 -	176.82 +	265.53 -	356.82 +
88.09 -	179.37 +	268.09 -	359.37 +
90.64 -		270.64 -	

DIVISION FACTOR = 88

90 SELECTED ORDINATES FOR
COS(30)

1.28 +	99.98 +	181.28 -	279.98 -
3.85 +	104.28 +	183.85 -	284.28 -
6.50 +	107.72 +	186.50 -	287.72 -
9.28 +	110.73 +	189.28 -	290.73 -
12.29 +	113.51 +	192.29 -	293.51 -
15.73 +	116.16 +	195.73 -	296.16 -
20.03 +	118.73 +	200.03 -	298.73 -
30.00	121.28 +	210.00	301.28 -
39.98 -	123.85 +	219.98 +	303.85 -
44.28 -	126.50 +	224.28 +	306.50 -
47.72 -	129.28 +	227.72 +	309.28 -
50.73 -	132.29 +	230.73 +	312.29 -
53.51 -	135.73 +	233.51 +	315.73 -
56.16 -	140.03 +	236.16 +	320.03 -
58.73 -	150.00	238.73 +	330.00
61.28 -	159.98 -	241.28 +	339.98 +
63.85 -	164.28 -	243.85 +	344.28 +
66.50 -	167.72 -	246.50 +	347.72 +
69.28 -	170.73 -	249.28 +	350.73 +
72.29 -	173.51 -	252.29 +	353.51 +
75.73 -	176.16 -	255.73 +	356.16 +
80.03 -	178.73 -	260.03 +	358.73 +
90.00		270.00	

DIVISION FACTOR = 84

90 SELECTED ORDINATES FOR
COS(40)

0.32 +	91.60 +	180.32 +	271.60 +
2.89 +	94.20 +	182.89 +	274.20 +
5.55 +	96.96 +	185.55 +	276.96 +
8.44 +	100.04 +	188.44 +	280.04 +
11.80 +	103.83 +	191.80 +	283.83 +
16.42 +	112.50	196.42 +	292.50
28.59 -	121.18 -	208.59 -	301.18 -
33.21 -	124.97 -	213.21 -	304.97 -
36.57 -	128.05 -	216.57 -	308.05 -
39.46 -	130.81 -	219.46 -	310.81 -
42.12 -	133.41 -	222.12 -	313.41 -
44.69 -	135.96 -	224.69 -	315.96 -
47.24 -	138.54 -	227.24 -	318.54 -
49.87 -	141.25 -	229.87 -	321.25 -
52.69 -	144.22 -	232.69 -	324.22 -
55.89 -	147.77 -	235.89 -	327.77 -
60.02 -	153.22 -	240.02 -	333.22 -
71.79 0	164.99 +	251.79 +	344.99 +
77.24 +	169.12 +	257.24 +	349.12 +
80.79 +	172.32 +	260.79 +	352.32 +
83.76 +	175.14 +	263.76 +	355.14 +
86.47 +	177.77 +	266.47 +	357.77 +
89.05 +		269.05 +	

DIVISION FACTOR = 88

90 SELECTED ORDINATES FOR
COS(50)

1.28 +	97.79 -	181.28 +	277.79 -
3.90 +	101.26 -	183.90 +	281.26 -
6.75 +	104.11 -	186.75 +	284.11 -
10.22 +	106.73 -	190.22 +	286.73 -
18.00	109.28 +	198.00	289.28 +
25.79 -	111.90 +	205.79 -	291.90 +
29.26 -	114.75 +	209.26 -	294.75 +
32.11 -	118.22 +	212.11 -	298.22 +
34.73 -	126.00	214.73 -	306.00
37.28 +	133.79 -	217.28 +	313.79 -
39.90 +	137.26 -	219.90 +	317.26 -
42.75 +	140.11 -	222.75 +	320.11 -
46.22 +	142.73 -	226.22 +	322.73 -
54.00	145.28 0	234.00	325.28 +
61.79 -	147.90 +	241.79 -	327.90 +
65.26 -	150.75 +	245.26 -	330.75 +
68.11 -	154.22 +	248.11 -	334.22 +
70.73 -	162.00	250.73 -	342.00
73.28 +	169.79 -	253.28 +	349.79 -
75.90 +	173.26 -	255.90 +	353.26 -
78.75 0	176.11 -	258.75 +	356.11 -
82.22 +	178.73 -	262.22 +	358.73 -
90.00		270.00	

DIVISION FACTOR = 80

90 SELECTED ORDINATES FOR
COS(60)

0.64 +	91.93 -	180.64 +	271.93 -
3.25 +	94.64 -	183.25 +	274.64 -
6.15 +	97.87 -	186.15 +	277.87 -
10.02 +	105.00	190.02 +	285.00
19.99 -	112.14 +	199.99 -	292.14 +
23.86 -	115.37 +	203.86 -	295.37 +
26.76 -	118.08 +	206.76 -	298.08 +
29.37 -	120.64 +	209.37 -	300.64 +
31.93 -	123.25 +	211.93 -	303.25 +
34.64 -	126.15 +	214.64 -	306.15 +
37.87 -	130.02 +	217.87 -	310.02 +
45.00	139.99 -	225.00	319.99 -
52.14 +	143.86 -	232.14 +	323.86 -
55.37 +	146.76 -	235.37 +	326.76 -
58.08 +	149.37 -	238.08 +	329.37 -
60.64 +	151.93 -	240.64 +	331.93 -
63.25 +	154.64 -	243.25 +	334.64 -
66.15 +	157.87 -	246.15 +	337.87 -
70.02 +	165.00	250.02 +	345.00
79.99 -	172.14 +	259.99 -	352.14 0
83.86 -	175.37 +	263.86 -	355.37 +
86.76 -	178.08 +	266.76 -	358.08 +
89.37 -		269.37 -	

DIVISION FACTOR = 84

90 SELECTED ORDINATES FOR COS(70)				90 SELECTED ORDINATES FOR COS(80)			
1.28 +	96.64 +	181.28 -	276.64 -	0.80 +	92.10 +	180.80 +	272.10 +
3.98 +	99.69 +	183.98 -	279.69 -	3.48 +	95.02 +	183.48 +	275.02 +
7.30 +	102.32 +	187.30 -	282.32 -	6.92 +	101.25	186.92 +	281.25
16.34 -	104.88 +	196.34 +	284.88 -	15.59 -	107.49 -	195.59 -	287.49 -
20.45 -	107.68 +	200.45 +	287.68 -	19.03 -	110.41 -	199.03 -	290.41 -
23.32 -	111.44 +	203.32 +	291.44 -	21.71 -	112.98 -	201.71 -	292.98 -
25.90 -	120.68 -	205.90 +	300.68 +	24.27 -	115.63 -	204.27 -	295.63 -
28.50 -	124.19 -	208.50 +	304.19 +	27.11 -	118.89 -	207.11 -	298.89 -
31.45 -	126.93 -	211.45 +	306.93 +	31.61 -	127.50 +	211.61 -	307.50 +
36.13 -	129.49 -	216.13 +	309.49 +	39.56 +	131.16 +	219.56 +	311.16 +
44.70 +	132.14 -	224.70 -	312.14 +	42.57 +	133.89 +	222.57 +	313.89 +
47.87 +	135.31 -	227.87 -	315.31 +	45.16 +	136.45 +	225.16 +	316.45 +
50.52 +	143.88 +	230.52 -	323.88 -	47.78 +	139.22 +	227.78 +	319.22 +
53.08 +	148.56 +	233.08 -	328.56 -	50.90 +	143.21 +	230.90 +	323.21 +
55.82 +	151.51 +	235.82 -	331.51 -	59.30 -	151.61 -	239.30 -	331.61 -
59.33 +	154.11 +	239.33 -	334.11 -	63.29 -	154.73 -	243.29 -	334.73 -
68.57 -	156.69 +	248.57 +	336.69 -	66.06 -	157.35 -	246.06 -	337.35 -
72.33 -	159.56 +	252.33 +	339.56 -	68.62 -	159.94 -	248.62 -	339.94 -
75.13 -	163.67 +	255.13 +	343.67 -	71.35 -	162.95 -	251.35 -	342.95 -
77.69 -	172.71 -	257.69 +	352.71 +	75.01 -	170.90 +	255.01 -	350.90 +
80.32 -	176.03 -	260.32 +	356.03 +	83.62 +	175.40 +	263.62 +	355.40 +
83.37 -	178.73 -	263.37 +	358.73 +	86.88 +	178.24 +	266.88 +	358.24 +
90.00		270.00		89.53 +		269.53 +	

DIVISION FACTOR = 88

DIVISION FACTOR = 88

90 SELECTED ORDINATES FOR
COS(90)

1.29 +	95.91 -	181.29 -	275.91 +
4.10 +	98.72 -	184.10 -	278.72 +
10.00	101.29 -	190.00	281.29 +
15.91 -	104.10 -	195.91 +	284.10 +
18.72 -	110.00	198.72 +	290.00
21.29 -	115.91 +	201.29 +	295.91 -
24.10 -	118.72 +	204.10 +	298.72 -
30.00	121.29 +	210.00	301.29 -
35.91 +	124.10 +	215.91 -	304.10 -
38.72 +	130.00	218.72 -	310.00
41.29 +	135.91 -	221.29 -	315.91 +
44.10 +	138.72 -	224.10 -	318.72 +
50.00	141.29 +	230.00	321.29 +
55.91 -	144.10 -	235.91 +	324.10 +
58.72 -	150.00	238.72 +	330.00
61.29 -	155.91 +	241.29 +	335.91 -
64.10 -	158.72 +	244.10 +	338.72 -
70.00	161.29 +	250.00	341.29 -
75.91 +	164.10 +	255.91 -	344.10 -
78.72 +	170.00	258.72 -	350.00
81.29 +	175.91 -	261.29 -	355.91 +
84.10 +	178.72 -	264.10 -	358.72 +
90.00		270.00	

DIVISION FACTOR = 72

90 SELECTED ORDINATES FOR
COS(100)

1.95 +	93.38 -	181.95 +	273.38 -
5.11 +	99.00	185.11 +	279.00
12.90 -	104.63 +	192.90 -	284.63 +
16.06 -	107.37 +	196.06 -	287.37 +
18.64 -	109.95 +	198.64 -	289.95 +
21.38 -	113.11 +	201.38 -	293.11 +
27.00	120.90 -	207.00	300.90 -
32.63 +	124.06 -	212.63 +	304.06 -
35.37 +	126.64 -	215.37 +	306.64 -
37.95 +	129.38 -	217.95 +	309.38 -
41.11 +	135.00	221.11 +	315.00
48.90 -	140.63 +	228.90 -	320.63 +
52.06 -	143.37 +	232.06 -	323.37 +
54.64 -	145.95 +	234.64 -	325.95 +
57.38 -	149.11 +	237.38 -	329.11 +
63.00	156.90 -	243.00	336.90 -
68.63 +	160.06 -	248.63 +	340.06 -
71.37 +	162.64 -	251.37 +	342.64 -
73.95 +	165.38 -	253.95 +	345.38 -
77.11 +	171.00	257.11 +	351.00
84.90 -	176.63 +	264.90 -	356.63 +
88.06 -	179.37 +	268.06 -	359.37 +
90.64 -		270.64 -	

DIVISION FACTOR = 80

90 SELECTED ORDINATES FOR COS(110)				90 SELECTED ORDINATES FOR COS(120)			
1.29 +	95.39 +	181.29 -	275.39 -	0.97 +	92.32 +	180.97 +	272.32 +
4.29 +	98.07 +	184.29 -	278.07 -	3.94 +	97.50	183.94 +	277.50
11.73 -	100.72 +	191.73 +	280.72 -	11.07 -	102.69 -	191.07 -	282.69 -
14.84 -	104.81 +	194.84 +	284.81 -	14.04 -	105.32 -	194.04 -	285.32 -
17.42 -	111.48 -	197.42 +	291.48 +	16.63 -	108.08 -	196.63 -	288.08 -
20.33 -	114.20 -	200.33 +	294.20 +	20.01 -	115.00 +	200.01 -	295.00 +
27.70 +	116.82 -	207.70 -	296.82 +	26.93 +	118.38 +	206.93 +	298.38 +
30.96 +	120.52 -	210.96 -	300.52 +	29.69 +	120.97 +	209.69 +	300.97 +
33.55 +	127.56 +	213.55 -	307.56 -	32.32 +	123.94 +	212.32 +	303.94 +
36.38 +	130.33 +	216.38 -	310.33 -	37.50	131.07 -	217.50	311.07 -
43.63 -	132.93 +	223.63 +	312.93 -	42.69 -	134.04 -	222.69 -	314.04 -
47.08 -	136.38 +	227.08 +	316.38 -	45.32 -	136.63 -	225.32 -	316.63 -
49.68 -	143.63 -	229.68 +	323.63 +	48.08 -	140.01 -	228.08 -	320.01 -
52.45 -	146.46 -	232.45 +	326.46 +	55.00 +	146.93 +	235.00 +	326.93 +
59.49 +	149.05 -	239.49 -	329.05 +	58.38 +	149.69 +	238.38 +	329.69 +
63.19 +	152.31 -	243.19 -	332.31 +	60.97 +	152.32 +	240.97 +	332.32 +
65.81 +	159.68 +	245.81 -	339.68 -	63.94 +	157.50	243.94 +	337.50
68.53 +	162.59 +	248.53 -	342.59 -	71.07 -	162.69 -	251.07 -	342.69 -
75.20 -	165.17 +	255.20 +	345.17 -	74.04 -	165.32 -	254.04 -	345.32 -
79.29 -	168.28 +	259.29 +	348.28 -	76.63 -	168.08 -	256.63 -	348.08 -
81.94 -	175.72 -	261.94 +	355.72 +	80.01 -	175.00 +	260.01 -	355.00 +
84.62 -	178.72 -	264.62 +	358.72 +	86.93 +	178.38 +	266.93 +	358.38 +
90.00		270.00		89.69 +		269.69 +	

DIVISION FACTOR = 88

DIVISION FACTOR = 84

90 SELECTED ORDINATES FOR
COS(130)

1.30 +	95.01 -	181.30 -	275.01 +
4.63 +	97.62 -	184.63 -	277.62 +
11.26 -	100.56 -	191.26 +	280.56 +
13.95 -	107.42 +	193.95 +	287.42 -
16.69 -	110.28 +	196.69 +	290.28 -
23.44 +	112.91 +	203.44 -	292.91 -
26.61 +	119.02 -	206.61 -	299.02 +
29.20 +	122.91 -	209.20 -	302.91 +
32.75 +	125.51 -	212.75 -	305.51 +
39.18 -	128.55 -	219.18 +	308.55 +
41.84 -	135.38 +	221.84 +	315.38 -
44.63 -	138.17 +	224.63 +	318.17 -
51.46 +	140.83 +	231.46 -	320.83 -
54.50 +	147.26 -	234.50 -	327.26 +
57.10 +	150.81 -	237.10 -	330.81 +
60.99 +	153.40 -	240.99 -	333.40 +
67.10 -	156.57 -	247.10 +	336.57 +
69.73 -	163.32 +	249.73 +	343.32 -
72.59 -	166.06 +	252.59 +	346.06 -
79.45 +	168.75 +	259.45 -	348.75 -
82.39 +	175.38 -	262.39 -	355.38 +
85.00 +	178.71 -	265.00 -	358.71 +
90.00		270.00	

DIVISION FACTOR = 88

90 SELECTED ORDINATES FOR
COS(140)

0.28 +	91.59 -	180.28 +	271.59 -
3.12 +	96.43	183.12 +	276.43
9.75 -	101.28 +	189.75 -	281.28 +
12.59 -	103.87 +	192.59 -	283.87 +
15.27 -	107.15 +	195.27 -	287.15 +
21.77 +	113.53 -	201.77 +	293.53 -
24.90 +	116.18 -	204.90 +	296.18 -
27.50 +	119.09 -	207.50 +	299.09 -
33.37 -	125.71 +	213.37 -	305.71 +
37.19 -	128.49 +	217.19 -	308.49 +
39.78 -	131.21 +	219.78 -	311.21 +
43.27 -	137.79 -	223.27 -	317.79 -
49.45 +	140.80 -	229.45 +	320.80 -
52.07 +	143.42 -	232.07 +	323.42 -
55.08 +	149.60 +	235.08 +	329.60 +
61.66 -	153.09 +	241.66 -	333.09 +
64.38 -	155.68 +	244.38	335.68 +
67.16 -	159.50 +	247.1	339.50 +
73.78 +	165.37 -	253.78 +	345.37 -
76.69 +	167.97 -	256.69 +	347.97 -
79.34 +	171.10 -	259.34 +	351.10 -
85.72 -	177.60 +	265.72 -	357.60 +
89.00 -		269.00 -	

DIVISION FACTOR = 88

90 SELECTED ORDINATES FOR
COS(150)

1.30 +	94.71 +	181.30 -	274.71 -
6.00	97.30 +	186.00	277.30 -
10.71 -	102.00	190.71 +	282.00
13.30 -	106.71 -	193.30 +	286.71 +
18.00	109.30 -	198.00	289.30 +
22.71 +	114.00	202.71 -	294.00
25.30 +	118.71 +	205.30 -	298.71 -
30.00	121.30 +	210.00	301.30 -
34.71 -	126.00	214.71 +	306.00
37.30 -	130.71 -	217.30 +	310.71 +
42.00	133.30 -	222.00	313.30 +
46.71 +	138.00	226.71 -	318.00
49.30 +	142.71 +	229.30 -	322.71 -
54.00	145.30 +	234.00	325.30 -
58.71 -	150.00	238.71 +	330.00
61.30 -	154.71 -	241.30 +	334.71 +
66.00	157.30 -	246.00	337.30 +
70.71 +	162.00	250.71 -	342.00
73.30 +	166.71 +	253.30 -	346.71 -
78.00	169.30 +	258.00	349.30 -
82.71 -	174.00	262.71 +	354.00
85.30 -	178.71 -	265.30 +	358.71 +
90.00		270.00	

DIVISION FACTOR = 60

90 SELECTED ORDINATES FOR
COS(160)

2.52 +	95.63	182.52 +	275.63
8.75 -	100.21 -	188.75 -	280.21 -
11.50 -	102.82 -	191.50 -	282.82 -
14.45 -	108.76 +	194.45 -	288.76 +
20.59 +	111.95 +	200.59 +	291.95 +
23.23 +	114.62 +	203.23 +	294.62 +
26.61 +	120.81 -	206.61 +	300.81 -
32.37 -	123.68 -	212.37 -	303.68 -
34.98 -	126.48 -	214.98 -	306.48 -
40.46 +	132.71 +	220.46 +	312.71 +
44.13 +	135.41 +	224.13 +	315.41 +
46.75 +	138.47 +	226.75 +	318.47 +
52.80 -	144.52 -	232.80 -	324.52 -
55.86 -	147.14 -	235.86 -	327.14 -
58.56 -	150.81 -	238.56 -	330.81 -
64.79 +	156.29 +	244.79 +	336.29 +
67.59 +	158.90 +	247.59 +	338.90 +
70.46 +	164.66 -	250.46 +	344.66 -
76.65 -	168.04 -	256.65 -	348.04 -
79.32 -	170.68 -	259.32 -	350.68 -
82.51 -	176.82 +	262.51 -	356.82 +
88.45 +	179.77 +	268.45 +	359.77 +
91.06 +		271.06 +	

DIVISION FACTOR = 88

90 SELECTED ORDINATES FOR
COS(170)

1.31 +	94.47 -	181.31 -	274.47 +
7.06 -	97.11 -	187.06 +	277.11 +
10.22 -	103.11 +	190.22 +	284.11 -
12.95 -	105.96 +	192.95 +	285.96 -
19.01 +	108.89 +	199.01 -	288.89 -
21.71 +	114.84 -	201.71 -	294.84 +
25.04 +	117.46 -	205.04 -	297.46 +
30.62 -	122.78 +	210.62 +	302.78 -
33.24 -	126.39 +	213.24 +	306.39 -
39.10 +	129.05 +	219.10 -	309.05 -
42.13 +	135.09 -	222.13 -	315.09 +
44.92 +	137.88 -	224.92 -	317.88 +
50.96 -	140.91 -	230.96 +	320.91 +
53.62 -	146.77 +	233.62 +	326.77 -
57.23 -	149.39 +	237.23 +	329.39 -
62.55 +	154.97 -	242.55 -	334.97 +
65.17 +	158.30 -	245.17 -	338.30 +
71.12 -	161.00 -	251.12 +	341.00 +
74.05 -	167.06 +	254.05 +	347.06 -
76.90 -	169.79 +	256.90 +	349.79 -
82.90 +	172.95 +	262.90 -	352.95 -
85.54 +	178.70 -	265.54 -	358.70 +
90.00		270.00	

DIVISION FACTOR = 88

90 SELECTED ORDINATES FOR
COS(180)

2.05 +	95.00	182.05 +	275.00
7.96 -	99.36 +	187.96 -	279.36 +
10.65 -	102.05 +	190.65 -	282.05 +
15.00	107.96 -	195.00	287.96 -
19.36 +	110.65 -	199.36 +	290.65 -
22.05 +	115.00	202.05 +	295.00
27.96 -	119.36 +	207.96 -	299.36 +
30.65 -	122.05 +	210.65 -	302.05 +
35.00	127.96 -	215.00	307.96 -
39.36 +	130.65 -	219.36 +	310.65 -
42.05 +	135.00	222.05 +	315.00
47.96 -	139.36 +	227.96 -	319.36 +
50.65 -	142.05 +	230.65 -	322.05 +
55.00	147.96 -	235.00	327.96 -
59.36 +	150.65 -	239.36 +	330.65 -
62.05 +	155.00	242.05 +	335.00
67.96 -	159.36 +	247.96 -	339.36 +
70.65 -	162.05 +	250.65 -	342.05 +
75.00	167.96 -	255.00	347.96 -
79.36 +	170.65 -	259.36 +	350.65 -
82.05 +	175.00	262.05 +	355.00
87.96 -	179.36 +	267.96 -	359.36 +
90.65 -		270.65 -	

DIVISION FACTOR = 72

90 SELECTED ORDINATES FOR
COS(190)

1.32 +	94.27 +	181.32 -	274.27 -
7.00 -	97.03 +	187.00 +	277.03 -
9.81 -	102.75 -	189.81 +	282.75 +
13.31 -	105.38 -	193.31 +	285.38 +
18.35 +	111.00 +	198.35 -	291.00 -
21.06 +	113.89 +	201.06 -	293.89 -
26.81 -	117.14 +	206.81 +	297.14 -
29.45 -	122.42 -	209.45 +	302.42 +
34.99 +	125.10 -	214.99 -	305.10 +
37.97 +	130.86 +	217.97 -	310.86 -
41.06 +	133.52 +	221.06 -	313.52 -
46.49 -	138.95 -	226.49 +	318.95 +
49.15 -	142.04 -	229.15 +	322.04 +
54.91 +	145.02 -	234.91 -	325.02 +
57.59 +	150.56 +	237.59 -	330.56 -
62.87 -	153.20 +	242.87 +	333.20 -
66.12 -	158.95 -	246.12 +	338.95 +
69.01 -	161.66 -	249.01 +	341.66 +
74.63 +	166.70 +	254.63 -	346.70 -
77.26 +	170.20 +	257.26 -	350.20 -
82.98 -	173.01 +	262.98 +	353.01 -
85.74 -	178.69 -	265.74 +	358.69 +
90.00		270.00	

DIVISION FACTOR = 88

90 SELECTED ORDINATES FOR
COS(200)

1.69 +	94.50	181.69 +	274.50
7.32 -	98.69 -	187.32 -	278.69 -
9.98 -	101.56 -	189.98 -	281.56 -
15.45 +	107.03 +	195.45 +	287.03 +
18.32 +	109.69 +	198.32 +	289.69 +
22.50	115.32 -	202.50	295.32 -
26.69 -	117.98 -	206.69 -	297.98 -
29.56 -	123.45 +	209.56 -	303.45 +
35.03 +	126.32 +	215.03 +	306.32 +
37.69 +	130.50	217.69 +	310.50
43.32 -	134.69 -	223.32 -	314.69 -
45.98 -	137.56 -	225.98 -	317.56 -
51.45 +	143.03 +	231.45 +	323.03 +
54.32 +	145.69 +	234.32 +	325.69 +
58.50	151.32 -	238.50	331.32 -
62.69 -	153.98 -	242.69 -	333.98 -
65.56 -	159.45 +	245.56 -	339.45 +
71.03 +	162.32 +	251.03 +	342.32 +
73.69 +	166.50	253.69 +	346.50
79.32 -	170.69 -	259.32 -	350.69 -
81.98 -	173.56 -	261.98 -	353.56 -
87.45 +	179.03 +	267.45 +	359.03 +
90.32 +		270.32 +	

DIVISION FACTOR = 80

90 SELECTED ORDINATES FOR
COS(210)

1.33 +	94.11 -	181.33 -	274.11 +
6.82 -	97.15 -	186.82 +	277.15 +
9.50 -	102.31 +	189.50 +	282.31 -
14.90 +	105.11 +	194.90 -	285.11 -
17.70 +	110.51 -	197.70 -	290.51 +
22.86 -	113.19 -	202.86 +	293.19 +
25.90 -	118.68 +	205.90 +	298.68 -
30.00	121.33 +	210.00	301.33 -
34.11 +	126.82 -	214.11 -	306.82 +
37.15 +	129.50 -	217.15 -	309.50 +
42.31 -	134.90 +	222.31 +	314.90 -
45.11 -	137.70 +	225.11 +	317.70 -
50.51 +	142.86 -	230.51 -	322.86 +
53.19 +	145.90 -	233.19 -	325.90 +
58.68 -	150.00	238.68 +	330.00
61.33 -	154.11 +	241.33 +	334.11 -
66.82 +	157.15 +	246.82 -	337.15 -
69.50 +	162.31 -	249.50 -	342.31 +
74.90 -	165.11 -	254.90 +	345.11 +
77.70 -	170.51 +	257.70 +	350.51 -
82.86 +	173.19 +	262.86 -	353.19 -
85.90 +	178.68 -	265.90 -	358.68 +
90.00		270.00	

DIVISION FACTOR = 84